

9th Annual

9th and 10th Mathematics Exam

Sponsored by: ΠΜΕ

1. Which set includes all values of x which satisfy the following equation?

$$2x - 3 > 7 - x$$

- A. $\{x: x > 4\}$
- B. $\{x: x < 10/3\}$
- C. $\{x: x = 10/3\}$
- D. $\{x: x > 10/3\}$

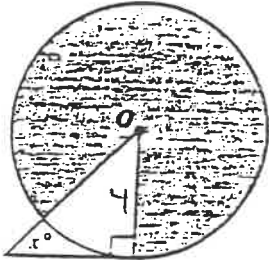
2.) How many of the first 100 positive integers are evenly divisible by each of the numbers 2, 3, 4, 5, 6?

- A. 0
- B. 1
- C. 2
- D. 3

3. Exactly three years before the year in which Andy was born, the year was $1988 - x$. In 2003, Andy will turn 20. Which of the following equations can be used to determine the value of x .

- A. $2003 - 1988 = x + 3 + 20$
- B. $1988 - x - 3 = 2003 - 20$
- C. $1988 - x + 3 = 2003 - 20$
- D. $2003 - 1988 = x + 3 - 20$

4. The circle with center O has a radius of 4. If the area of the shaded region is 14π , what is the value of x ?



- A. 75
B. 60
C. 55
D. 45
5. $\log_4 64 - \log_2 64 =$
- A. $\log_2 64$
B. -3
C. 9
D. -16

6. Bryan wants to write code words. How many five-letter code words can he possibly make using the letters in FRANK, if the same letter may not be repeated in a single code word?

- A. 25
B. 120
C. 325
D. 5^5

7. The number of pigs and horses on Katie's farm are such that the ratio of their difference and sum is 1:7. Meanwhile, the ratio of their sum and product is 7:24. How many total pigs and horses does Katie have?

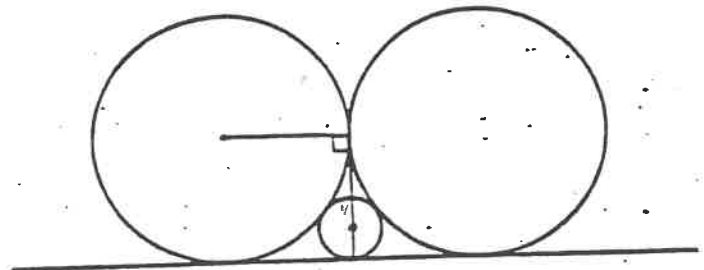
- A. 7
- B. 48
- C. 28
- D. 14

8. Josh makes a cube by soldering together 12 pieces of wire, each 3" long. Josh turns into a fly and lands at one vertex of the cube, then walks along its edges. What is the greatest distance Josh can travel before arriving at a vertex a second time without retracing his footsteps?

- A. 18"
- B. 21"
- C. 24"
- D. 27"

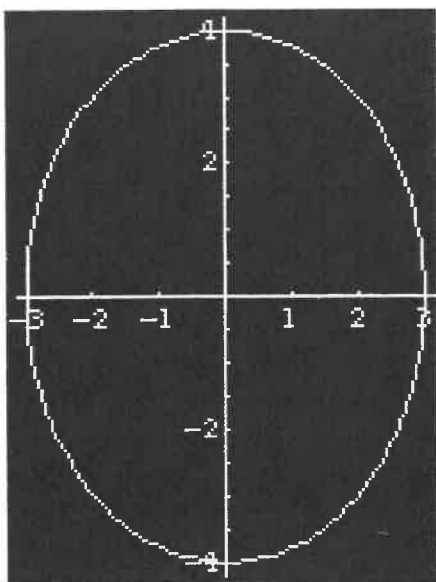
9. On the same side of a straight line three circles are drawn. One has a radius of 4", and the other two have equal areas. Each circle is tangent to the other two and to the line. The radius, in inches, of the equal circles is:

- A. 16
- B. 18
- C. 20
- D. 24

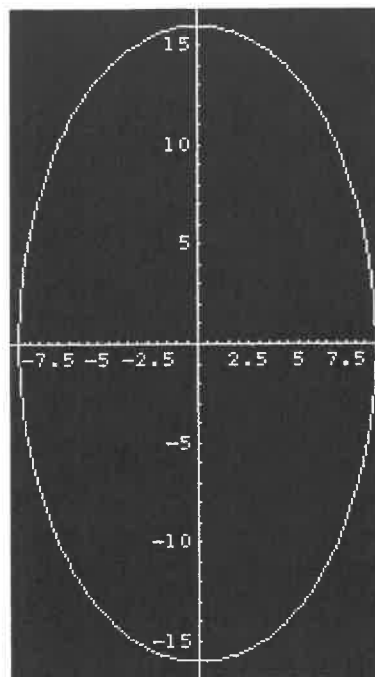


10. Which of the following is the graph of $\frac{x^2}{9} + \frac{y^2}{16} = 1$?

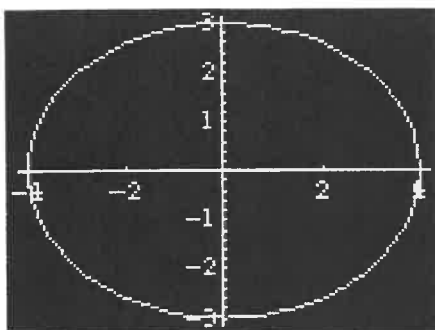
A.



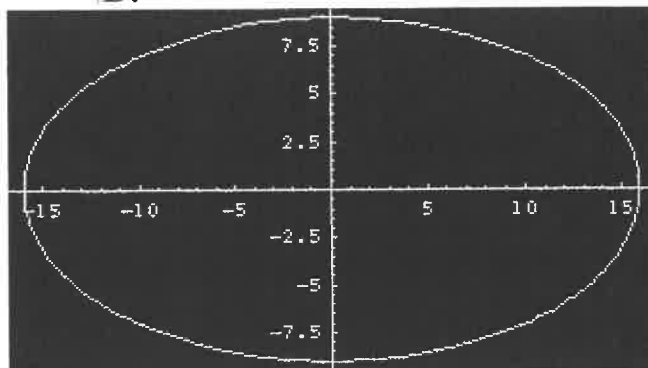
B.



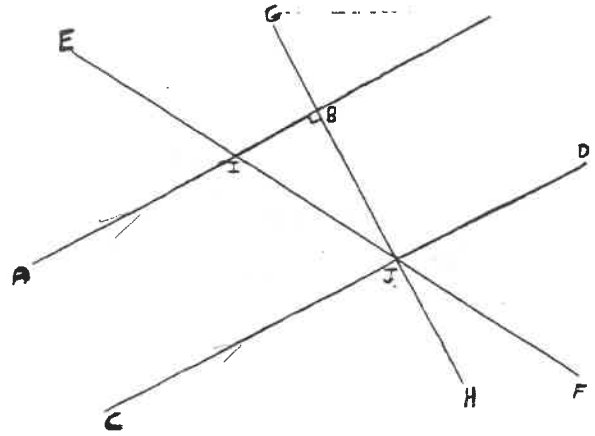
C.



D.



11. $\overline{AB} \parallel \overline{CD}$, $\overline{GH} \perp \overline{AB}$,
 $\angle DJF = 60^\circ$ and $\overline{IJ} = 20\text{m}$.
 Find \overline{BJ} .



- A. 10m
 B. $10\sqrt{2}$ m
 C. $10\sqrt{3}$ m
 D. 20m

12. Given:

$$\tan \Theta = \frac{\sin \Theta}{\cos \Theta}; \sin^2 \Theta + \cos^2 \Theta = 1,$$

$$\text{Simplify } \left(\frac{1}{\tan^2 \Theta} \right) (1 - \cos^2 \Theta).$$

- A. 1
 B. $\sin^2 \Theta$
 C. $1 - \sin^2 \Theta$
 D. $\frac{\sin^4 \Theta}{\cos^2 \Theta}$

13.
$$\frac{\left[2^{\left(\frac{n}{2}+2\right)}\right]^2 - 2(2^n)}{2(2^{n+3})}$$

A: $2^{n+1} - \frac{1}{8}$

B: $1 - 2^n$

C: $\frac{7}{16}$

D: $\frac{7}{8}$

14. In a haunted house, Bob the Frog sits on a stand with his mouth 5 feet 6 inches about the ground. It is dark, and as people round a corner, they pass 6 inches in front of his nose. Ed sticks out his tongue and gets them on the nose. If the tallest person he can hit on the target has his or her nose 6 feet 6 inches above the ground, how long is Bob's tongue?

A. 15 inches

B. $6\sqrt{2}$ inches

C. $6\sqrt{3}$ inches

D. $6\sqrt{5}$ inches

15.
$$\frac{1}{2 + \frac{1}{2 + \frac{1}{2}}} =$$

- A. 1/3
- B. 2/5
- C. 2/9
- D. 5/12

16. An ancient Warlock named Andy is twice as old as his son. 25 years ago, their age ratio strangely enough was 3 to 1. When does (did) Andy celebrate his centenary.

- A. 5 years ago
- B. when his son is 50
- C. 20 years ago
- D. when his son is 90

17. If $[(2^2)^3]^{x+1} = 4^6$, find x.

- A. 1
- B. 3
- C. 0
- D. 1/5

18. How much less is the area of a rectangular field 60 meters by 40 meters than that of a square field with the same perimeter?

- A. 10 sq. m
- B. 100 sq. m
- C. 1000 sq. m
- D. They're equal.

19. A rectangular lot 30 feet by 40 feet is surrounded on all four sides by a concrete walk 5 feet wide. Find the area of the surface of the walk.

- A. 800 sq. ft.
- B. 700 sq. ft.
- C. 375 sq. ft.
- D. 350 sq. ft.

20) An isosceles triangle and a square have identical areas. Express the altitude h as a function of the base s .

- A. $h = s^2$
- B. $h = s/2$
- C. $h = 2s$
- D. $h = s^2/2$

21. What is the distance between the endpoints of the diagonal of a square with side of length x ?

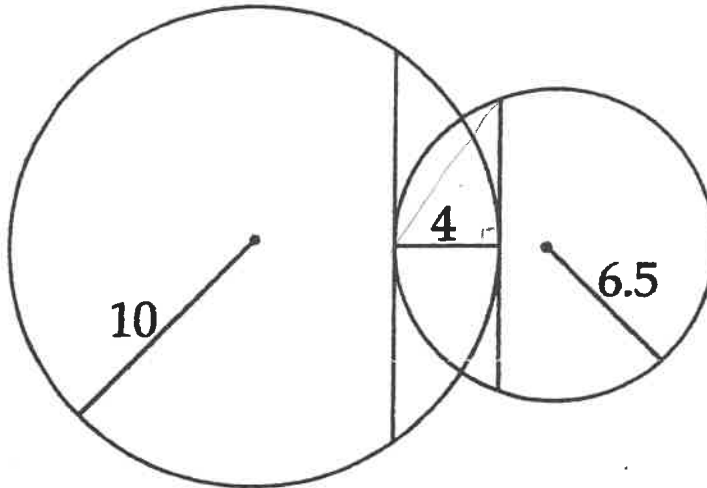
- A. $2x$
- B. x^2
- C. $x\sqrt{2}$
- D. $x\sqrt{3}$

22. What is the sum of the digits of the next term in this sequence?

-1, 1, 7, 25, ...

- A. 13
- B. 14
- C. 15
- D. 16

23. Circles with 20 cm and 13 cm diameters overlap 4 cm. Find the lengths of the tangent cords.



- A. 6.5 cm and 10 cm
- B. 12 cm and 16 cm
- C. 8 cm and 12 cm
- D. 9 cm and 12 cm

24. Ed owns a baseball team. He has \$20 million budgeted to pay the salaries of his 25 players. All of the salaries must be multiples of \$100,000 and the entire budget must be spent. Each player must receive at least \$100,000. 15 players must each receive at least \$200,000. The shortstop must receive at least \$1 million and the pitcher at least \$1.5 million. What is the maximum possible salary, in millions, that Ed can pay one of his players?

- A. \$15.2
- B. \$13.9
- C. \$13.7
- D. \$15.4

25. Telephone numbers in The United States have ten digits as follows: 3 digit area code in which the first digit is not 0 or 1, and the second digit must be 0 or 1. 3 digit exchange in which the first and second digit are not 0 or 1. 4 digit line which is not all 0's.

If there are 5 area codes in Illinois, how many phone numbers can Illinois have without adding another area code?

- A. 32,000,000
- B. 31,996,800
- C. 31,993,600
- D. 49,995,000

Test 11/12 Source List

(1) Problems: 1,2,4,13

Source: The Mathematical Association of America. *The MAA Problem Book I* and *The MAA Problem Book IV*.

(2) Problem: 25.

Source: Ritger, Andy. Past Student. Illinois Wesleyan University, Bloomington, IL. 1996

(3) Problems: 1,5,6,11.

Source: Votava, Micheal. Past Student. Illinois Wesleyan University, Bloomington, IL. 1996

(4) Problem: 8,24.

Source: Problem submitted by Rudd Crawford's Introduction to Mathematical Education (Math 211) class at Oberlin College. "March Calendar". *The Mathematics Teacher* 88 (March 1995).

(5) Problem: 12

Source: Roush, Brett. Past Student. Illinois Wesleyan University, Bloomington, IL. 1996.

(6) Problem: 9

Source: Problem submitted by Nathaniel Bates and Sanderson Smith. "February Calendar." *The Mathematics Teacher* 88 (February 1995)

(7) Problem: 14,16

Source: Problem submitted by Pauline Jacqueline McDonald. "October Calendar." *The Mathematics Teacher* 87 (October 1994)

(8) Problem: 7

Source: Problem submitted by Agnes Tuska.
"February Calendar." *The Mathematics Teacher* 88 (February 1995)

(9) Problems: 15,18,19,22

Source: Conrad, Steven, and Daniel Flegler. *The 1st Elementary Grades Math League Problem Book*. United States: Math League Press, 1986.

(10) Problems: 17

Source: Illinois Council of Teachers of Mathematics. "Regional Math Contest." CAN Insurance Companies: United States. 7 March 1992: 33.

(11) Problems: 20,23

Source: Problems Submitted by James Varnadore. "May Calendar." *The Mathematics Teacher* 87 (May 1994)

(12) Problems: 21

Source: Swichtenberg, Glenn. Past Dean of Student Affairs. Illinois Wesleyan University. Bloomington, Illinois 1995.