This is an individual test. No outside resources are allowed, not even calculators. Answer every question to the best of your ability, and circle your answer choice.

1. A bag contains red and blue marbles. When two marbles are drawn, the probability that they are both red is equal to the probability they are both blue. The probability that one of each color is drawn is $\frac{4}{7}$. Find the total number of marbles in the bag.

   (a) 7

   (b) 9

   (c) 10

   (d) 8

2. Let a, b and c be real numbers, and let $P(x) = ax^9 + bx^5 + cx + 3$. If $P(-5) = 17$, find $P(5)$.

   (a) -17

   (b) -11

   (c) 14

   (d) 17

3. What is the area (in square units) of the region in the first quadrant defined by $18 \leq x + y \leq 20$?

   (a) 38

   (b) 40

   (c) 42

   (d) 44

4. How many real number solutions does the equation $5\sqrt{x} = 6 - x$ have?

   (a) 0

   (b) 1

   (c) 2

   (d) 3

5. Let a and b be the two roots of the equation $x^2 + 3x - 3 = 0$. Evaluate the value $a^2 + b^2$.

   (a) 9

   (b) 10

   (c) 12

   (d) 15

6. Let h and k be the roots of the equation $2x^2 - 9x + c = 0$. If $4hk = 8$, find $h + k + c$.

   (a) 20

   (b) 1

   (c) 10

   (d) 11

7. Which of the following is not a root for the polynomial $x^3 + 2x^2 - 35x$?

   (a) 0

   (b) 2

   (c) 5

   (d) 7

8. Given that a function and its inverse are defined over the Real numbers, and the $f(3) = 6$, $f(6) = -1$, and $f(-1) = 0$, what is $f^{-1}(-1)$?

   (a) 6

   (b) 0

   (c) 3

   (d) $(-\infty, \infty)$

9. Solve for x and y: $\begin{bmatrix} 4 & x \\ 1 & 0 \end{bmatrix} \begin{bmatrix} y & 3 \\ -1 & -3 \end{bmatrix} = \begin{bmatrix} 3 & 9 \\ 1 & 3 \end{bmatrix}$

   (a) (1, 1)

   (b) (1, 3)

   (c) (1, 2)

   (d) (2, 1)

10. The graph of $f(x) = 3 + \ln x - 2$ can be determined by one or more of the following operations on the function:

    i. shift the graph of $g(x) = 3 + \ln x$ two units to the right
    ii. shift the graph of $g(x) = 3 + \ln x$ two units to the left
    iii. shift the graph of $h(x) = \ln x$ two units to the right and 3 units up
    iv. shift the graph of $h(x) = \ln x$ two units to the right and 3 units down

    (a) i only
    (b) iii only
    (c) i and iii
    (d) ii and iv

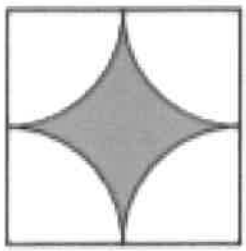
11. Given that i = $\sqrt{-1}$, evaluate $\sum_{k=0}^{2022} i^k$.

    (a) i
    (b) -i
    (c) 0
    (d) 1 + i

12. Suppose f(0) = $\sqrt{17}$. Evaluate $e^{\ln(f(0))}$.

    (a) 61.7507
    (b) $\sqrt{17}$
    (c) 1.4166
    (d) 1

13. An 8-foot by 10-foot floor is tiled with square tiles of size 1 foot by 1 foot. Each tile has a pattern consisting of four white quarter circles of radius 1/2 foot centered at each corner of the tile. The remaining portion of the tile is shaded. How many square feet of the floor are shaded?



    (a) 60 - 20π
    (b) 80 - 20π
    (c) 60 + 20π
    (d) 80 + 20π

14. What is the largest closed interval $[a,b]$ on which the following equalities hold?

$$\begin{aligned}
\arcsin\sin x &= x \\
\sin\arcsin x &= x \\
\arccos\cos x &= x \\
\cos\arccos x &= x \\
\arctan\tan x &= x \\
\tan\arctan x &= x
\end{aligned}$$

(a) $[0,1]$

(b) $[-\pi,\pi]$

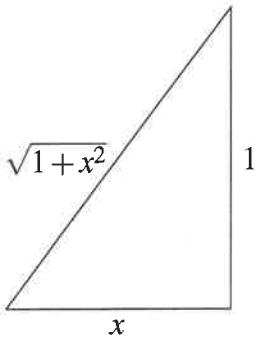
(c) $[-\frac{\pi}{2},\frac{\pi}{2}]$

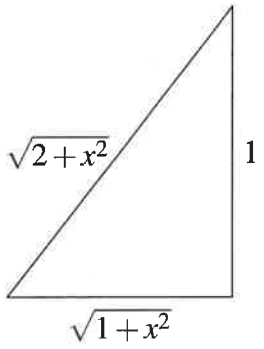
(d) $[-\infty,\infty]$

15. What is the algebraic representation in terms of x for the following expression?

$$\cos\left(\tan^{-1}\left(\sin\left(\cot^{-1}(x)\right)\right)\right)$$

Hint: You might find the following triangles helpful:





(a) x

(b) $\sqrt{\frac{2+x^2}{1+x^2}}$

(c) $\frac{1}{\sqrt{1+x^2}}$

(d) $\frac{1}{\sqrt{2+x^2}}$

16. If $2 = x^{x^{x^{x^{\cdots}}}}$ (an infinite tower of exponents), then x is

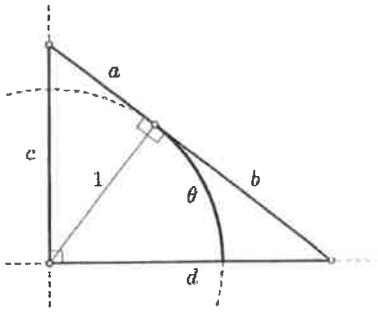
(a) $2^{1/2}$

(b) $e^{1/2}$

(c) $1/e^2$

(d) $2/e$

17. An arc length θ is measured on the unit circle and a tangent line is drawn where the arc terminates. Which of the indicated segments has length $\csc\theta$?



(a) a
(b) b
(c) c
(d) d

18. $f(x)$ be an odd function that has the property: $3f(x) - f(-x) = \sin x$. What is the value of $f(\pi/2)$?

(a) -1/2
(b) -1
(c) 1/4
(d) 1

19. Suppose that x and y are numbers such that $\sin(x+y) = 0.3$ and $\sin(x-y) = 0.5$. Then $\sin x * \cos y =$?

(a) 0.1
(b) 0.3
(c) 0.4
(d) 0.5

20. Given that $\log_{10}(\sin x) + \log_{10}(\cos x) = -1$ and $\log_{10}(\sin x + \cos x) = \frac{1}{2}\log_{10}(n-1)$, find n.

(a) 1.2
(b) 2
(c) 12
(d) 2.2

21. Suppose that $\cos(\alpha) = 0.7$. Which of the following is NOT true?

(a) $\cos(-\alpha) = 0.7$
(b) $\cos(\pi + \alpha) = 0.7$
(c) $\cos(2\pi + \alpha) = 0.7$
(d) $|\sin(\alpha)| = \sqrt{0.51}$