

This is a group test, so your entire team can work together. No outside resources are allowed, including calculators.

Answer the questions to the best of your ability. Write your arguments as clearly and concisely as possible. Your solutions will be graded based on your ability to explain your reasoning, as well as your final answer. Good luck!

Choose exactly 3 problems to submit solutions for.

1. Show that $p(x) = x^3 + x^2 - 2x - 1$ has only real roots.
2. Suppose you are hired to test and compare 7 different types of bubble tea. Design a schedule for testing them that meets the following conditions.
 - (a) Teas are tested in groups of 3.
 - (b) Every pair of distinct teas will be compared in exactly one of the groups of 3.
3. Prove that there are no consecutive primes beyond 4.
4. Show that at any gathering of exactly 6 people it must be true that either some 3 of them know each other, or some 3 of them are mutual strangers.
5. Recall that a regular tetrahedron is a pyramid with a triangular base, and a regular octahedron can be seen as two pyramids with square bases glued together at the base. Assuming that the regular tetrahedron and octahedron have equal side lengths, find the ratio of their volumes.