This is a group test, so your entire team can work together. No outside resources are allowed.

Answer the questions to the best of your ability. Write your arguments as clearly and concisely as possible. Your solutions will be graded based on your ability to explain your reasoning, as well as your final answer. Good luck!

Choose three problems for which you will submit solutions.

1. A *round robin* tournament is one in which each team plays against every other team exactly once. Suppose you have $2n$ teams where n is a positive integer. Develop a method for scheduling a round robin tournament for these $2n$ teams where each team plays exactly one game each day. Note that the tournament will take place over multiple days.

2. (written by Michael Ng) A bullet is formed by revolving the area bounded by the curve $y = \ln(x)$ from $x = 1$ to $x = e$ about the x-axis.

   It is then shot straight into a very thick wall (i.e. it does not pierce through the other side) making a closed cylindrical hole until it stops moving. Assume the bullet maintains its shape. Then it is carefully extracted without affecting the hole at all, leaving an empty hole with a pointy end where the bullet once was.

   The length of the entire hole is $e+1$. If the volume can be expressed as

   $$\alpha\pi e$$

   where α is a constant, find the value of α.

3. Show that $6^n + 4$ is divisible by 5 for every non-negative integer n.

4. Characterize all points on $y = \sin(x)$ for which their tangent line contains the point $(0, 1)$.