

**9<sup>th</sup> Annual**

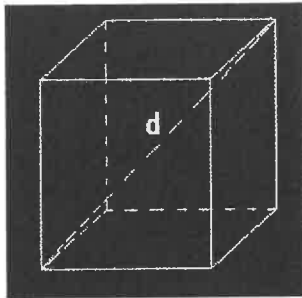
**11<sup>th</sup> and 12<sup>th</sup> Mathematics Exam**

**Sponsored by: TIME**

1. Exactly three years before the year in which Ed was born, the year was  $1980 - x$ . In 1995, Ed turned 20. Which of the following equations can be used to determine the value of  $x$ ?

- A.  $1995 - 1980 = x + 3 + 20$
- B.  $1995 - 1980 = x + 3 - 20$
- C.  $1980 - x + 3 = 1995 - 20$
- D.  $1980 - x - 3 = 1995 - 20$

2.



Define the length  $d$  of the internal diagonal of a cube as a function of the volume  $V$  of the cube. What does  $d = ?$

- A.  $V^{\frac{1}{3}} * \sqrt{3}$
- B.  $V * \sqrt{3}$
- C.  $V^{\frac{1}{2}} * \sqrt{3}$
- D.  $V^{\frac{2}{3}} * \sqrt{3}$

3. The half-life of polonium is 140 days. The sample Bryan has will not be useful once 90% of the radioactive nuclei originally present have disintegrated. Which answer is closest to the number of days that Bryan can use the polonium?

- A. 560 days
- B. 465 days
- C. 420 days
- D. 126 days

4. What is the largest possible value of  $y$  if

$$y = \sqrt{\frac{(1 + \cos[2x])}{8}} ?$$

- A.  $\infty$
- B.  $\frac{\sqrt{6}}{4}$
- C.  $\frac{1}{2}$
- D.  $\frac{\sqrt{2}}{4}$

5. Laura was on a train that, during the last 4 hours of its 400 km trip, averaged  $t$  km per hour. If the train completed the trip in 6 hours, what speed, in km per hour, did Laura's train average in the first 2 hours?

- A.  $\frac{200 + t}{2}$
- B.  $\frac{200 - t}{2}$
- C.  $200 + t$
- D.  $200 - 2t$

6. Bob's bamboo, which was growing perpendicular to the ground, was 18 cubits high but was broken by the wind. Its top touched the ground, which is level, 6 cubits from the root. What are the lengths of the segments of Bob's bamboo?

- A. 9, 9
- B. 10, 8
- C. 6, 12
- D. 6, 6

7. Sarah is standing perpendicular to the ground. Her shadow is 5 feet long. The angle formed by the line segments from her feet to her head and from her head to the tip of her shadow is  $51^\circ$ . If  $\text{Cot}[39^\circ] \approx 1.25$ , approximately how tall is Sarah in feet?

- A. 5
- B. 4
- C. 5.5
- D. 6.25

8. What is the set of all real solutions to the inequality:

$$|x - 1| + |x + 2| < 5$$

- A.  $\{x: -1 < x < 2\}$
- B.  $\{x: -3 < x < 2\}$
- C.  $\{x: -2 < x < 1\}$
- D.  $\{x: -2 < x < 5/2\}$

9. In our number system the base is 10. Laban decides he wants to count in base 4, so he counts as follows: 1,2,3,10, ... What is the number 20 in base 4?

- A. 50
- B. 38
- C. 120
- D. 110

10.  $x = \{(\ln 1)\left[\left(\frac{\log 25}{\log 8}\right) - \log 2\right]\} + \log .1 + \ln e$

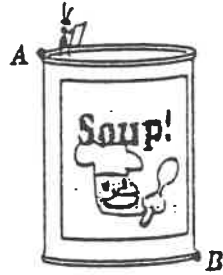
What is  $x$ ?

- A. 0
- B. 2
- C.  $-\log 2$
- D. 1

11. You are throwing darts at a circular dartboard. The innermost area is Mike's face, which is a perfect circle with a radius of 3 inches. The area around Mike's face is orange and bounded by a circle with a diameter 2 inches greater than the diameter of Mike's face. The outermost area is black and bounded by a circle with twice the diameter of Mike's face. What is the area, in square inches, of Mike's face plus the area of the black section? (All circles mentioned are concentric.)

- A.  $32\pi$
- B.  $29\pi$
- C.  $22.25\pi$
- D.  $20\pi$

12.



An open 20-cm-high cylindrical can has diameter  $20/\pi$  cm. Josh, who has recently become an ant, is at A on the top edge and walks to B, directly opposite A and on the bottom edge of the can. What is the length, in cm, of the shortest path Josh could have strolled?

- A.  $20\sqrt{5}$
- B.  $10\sqrt{5}$
- C.  $\sqrt{\frac{400\pi^2 + 400}{\pi^2}}$
- D. 30

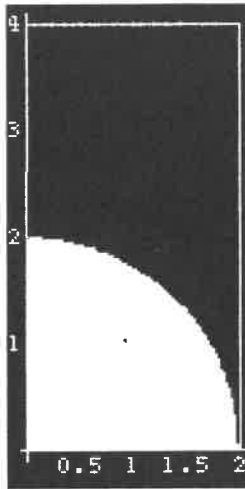
13. Andy drops a super bouncy ball from a height of 6 feet. Each bounce, the ball rises to a height  $2/3$  as high as on the previous bounce. After an infinite number of bounces up and down, what is the total vertical distance traveled by Andy's ball, in feet?

- A. 9
- B. 18
- C. 36
- D. 30

14.  $N = 8 \cdot 10 \cdot x^{-3}$  gives the number of people with income greater than  $x$  dollars. The lowest income, in dollars, of the wealthiest 800 individuals is at least:

- A.  $10^4$
- B.  $10^6$
- C.  $10^8$
- D.  $10^{12}$

15.



If Laura chooses a point within the rectangle formed by the origin,  $(2,0)$ ,  $(2, 4)$ , and  $(0, 4)$ , what is the probability that the point is in the area under the curve?

- A.  $\pi / 2$
- B.  $\pi / 8$
- C.  $\pi / 4$
- D.  $\pi$

16. All the streets in Square City run north-south and east-west. How many ways can Patrick walk from his house to Kelly's if she lives 6 blocks west and 4 blocks north? Keep in mind that Patrick is in love with

Kelly and thus always takes the shortest route, but he cannot cut across a block.

- A. 24
- B. 150
- C. 210
- D. 360

17. In a triangle with sides of lengths  $a$ ,  $b$ , and  $c$ ,  $(a + b + c)(a + b - c) = 3ab$ . By the law of cosines, what is the measure, in degrees, of the angle opposite the side with length  $c$ ?

- A. 60
- B. 45
- C. 30
- D. 15

18. John comes to Bloomington for a spatula conference. In Bloomington, the distance from the conference center to the airport is 10 miles. The distance from the conference center to the hotel is 10 miles as well. What is the probability that the hotel is closer to the airport than to the conference center?

- A.  $1/12$
- B.  $1/10$
- C.  $1/2$
- D.  $1/3$

19. Alicia tries to solve the following equations:

$$qx + (q - 1)y = 1$$

$$(q + 1)x - qy = 1$$

For which value of  $q$  can she find no solution for  $x$  and  $y$ ?

- A.  $\pm\sqrt{2}$
- B.  $\pm\frac{\sqrt{2}}{2}$
- C. 1
- D. -1

20. Steve owns a baseball team. He has \$20 million budgeted to pay the salaries of his 25 players. All of the salaries must be multiples of \$100,000 and the entire budget must be spent. Each player must receive at least \$100,000. 15 players must each receive at least \$200,000. The shortstop must receive at least \$1 million and the pitcher at least \$1.5 million. What is the maximum possible salary, in millions, that Steve can pay one of his players?

- A. \$15.2
- B. \$13.9
- C. \$13.7
- D. \$15.4

21. Find the following limit:

$$\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1}$$

- A. 0
- B.  $\frac{1}{2}$
- C. 2
- D.  $\infty$

22. Telephone numbers in The United States have ten digits as follows: a 3 digit area code in which the first digit is not 0 or 1, and the second digit must be 0 or 1; a 3 digit exchange in which the first and second digit are not 0 or 1; and a 4 digit line which is not all 0's.

*(area code) exchange - line*

If there are 5 area codes in Illinois, how many phone numbers can Illinois have without adding another area code?

- A. 32,000,000
- B. 31,996,800
- C. 28,800,000
- D. 49,995,000

23. Two bicycles are 20 miles apart and traveling towards each other at a constant speed of 10mph. A bird is flying above one of the bicycles towards the other at 15mph. When it reaches the other bicycle the bird immediately reverses direction, traveling until it reaches the first bicycle and again reverses direction. The bird continues flying in this manner until the bicycles meet. At that time how far has the bird traveled?

- A. cannot be determined from the given information
- B. 30 miles
- C. 35 miles
- D. 15 miles

24. The derivative,  $f'(x)$ , of a function  $f(x)$ , can be found by using the following definition:

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Using this definition, find the derivative of  $3x^2$ .

- A.  $6x$  .
- B.  $x^3$  .
- C.  $6x^2$  .
- D.  $x$  .

25. Find the number of integers in the first 999 natural numbers that can be divided evenly by neither 3 nor 5.

- A. 467
- B. 333
- C. 931
- D. 533

## Test 11/12 Source List

(1) Problems: 5,6,8,9,14,17,19,20.

Source: The Mathematical Association of America. *The MAA Problem Book I* and *The MAA Problem Book IV*.

(2) Problem: 1.

Source: Ritger, Andy. Past Student. Illinois Wesleyan University, Bloomington, IL. 1996

(3) Problems: 3,4,10.

Source: Votava, Micheal. Past Student. Illinois Wesleyan University, Bloomington, IL. 1996

(4) Problem: 2.

Source: Nelson, Bryan; Dockins, Josh; Rowland, Andrew; Scholten, Katie. Students. Illinois Wesleyan University. Bloomington, IL 2003

(5) Problem: 7,11,13.

Source: Roush, Brett. Past Student. Illinois Wesleyan University. Bloomington, IL. 1996.

(6) Problem: 12

Source: Problem submitted by Nathaniel Bates and Sanderson Smith. "February Calender." *The Mathematics Teacher* 88 (February 1995)

(7) Problem: 15

Source: Coates, Keith. Professor of Mathematics. Illinois Wesleyan University. Bloomington, Illinois. 1996.

(8) Problem: 16

Source: Problem submitted by Pauline Lowman and Joseph Stokes.  
"February Calender." *The Mathematics Teacher* 88 (February 1995)

(9) Problem: 18

Source: Problem submitted by Agnes Tuska.  
"February Calender." *The Mathematics Teacher* 88 (February 1995)

(10) Problems: 21,22,23,24,25

Source: 3<sup>rd</sup> Annual IWU Math Contest. 1997