10th Annual

11th and 12th Grade Mathematics Exam

Sponsored by:

ΠME

1. Exactly 2 years before the year in which Ed was born, the year was $1992 - x$. In 2002, Ed turned 16. Which of the following equations can be used to determine the value of x?

A. $2002 - 1992 = 2 + x - 16$
B. $1992 - x + 2 = 2002 - 16$
C. $1992 - x - 2 = 2002 - 16$
D. $2002 - x - 2 = 1992 + 16$

2. Laura was on a yacht that, during the last 3 hours of a 300 km trip, averaged y km/hr. If the yacht completed the trip in 5 hours, what speed, in km/hr, did Laura's yacht average in the first 2 hours of the trip?

A. $\frac{300 - 3t}{2}$

B. $\frac{300 - 2t}{3}$

C. $\frac{300 - t}{2}$

D. $300 + 2t$

3. Evaluate the following limit:

$$\lim_{x \to 3} \frac{x^2 - 4}{x - 2}$$

A. 0
B. ∞
C. 5
D. $\frac{3}{5}$

4. The derivative of a function f(x) can be found using the following definition:

$$f'(x) = \lim_{h \to o} \frac{f(x+h) - f(x)}{h}$$

Use this definition to find the derivative of $\frac{1}{4}x^8$

A. $\frac{1}{2}x^7$

B. $4x^7$

C. $2x^2$

D. $2x^7$

5. Two kids on big wheels are 100 yards apart on a football field. They ride towards each other at a constant rate of 5 yards/min. A butterfly begins flying directly above one of the kids, and flies towards the other kid at a constant rate of 15 yards/min. When it reaches the other kid, it immediately reverses direction, flying until it reaches the other kid and again reverses direction. The butterfly continues flying in this manner until the big wheels meet. At that time, how far has the butterfly traveled?

A. 200 yards
B. 175 yards
C. 150 yards
D. 100 yards

6. Evaluate: $\int (3x^3 + x^5 - 6x^8)dx$

A. $3x^2 + 5x^4 - 6x^7 + C$

B. $\frac{3x^2}{4} + \frac{x^6}{5} - \frac{2x^9}{3} + C$

C. $\frac{3x^4}{4} + \frac{x^6}{6} - \frac{2x^9}{3} + C$

D. $\frac{4x^4}{4} + 6x^6 - \frac{3x^9}{2} + C$

7. Jarkko's hockey stick was leaning against a wall before his big game. The stick makes a 37 degree angle with the wall and the top tip of the stick is 3.8 feet above the floor. Assume that the wall is perpendicular to the floor. If sec(37) = 1.25, how long is his hockey stick?

A. 3.04 feet
B. 4.75 feet
C. 5.25 feet
D. 4.5 feet

8. Find the value of $x^2 + y^2$:

$$\begin{bmatrix} 0 & 1 & 6 \\ 5 & x & 4 \\ 8 & 2 & 10 \end{bmatrix} \begin{bmatrix} 2 & y \\ 5 & 7 \\ \frac{1}{2} & 3 \end{bmatrix} = \begin{bmatrix} 8 & 25 \\ 27 & 53 \\ 31 & 76 \end{bmatrix}$$

A. 69
B. 34
C. 25
D. 41

9. Let a*b = $\frac{2b(a+b)}{3a^2b}$

Calculate $(2+3i)*(3+2i)$

A. $\frac{10+50i}{-105+36i}$

B. $\frac{50+10i}{105-36i}$

C. $\frac{10-50i}{-105-36i}$

D. $\frac{50+10i}{36-105i}$

10. Johnny is competing in a state swim competition. How fast will Johnny have to swim in meters/second to beat the state record of 2 minutes in the 200m fly?

A. 100 m/s
B. 0.6 m/s
C. 1 m/s
D. 1.67 m/s

11. If $x^3y+2x=3$ and $\frac{dx}{dy}=7$, solve for $\frac{dy}{dx}$.

A. $\frac{x^3y+2}{3x^2}$

B. $\frac{21x^2y+14}{x^3}$

C. $\frac{21x^2y-14}{3x^2}$

D. $\frac{14}{21x^2}$

12. Allison is standing against a wall and she is playing with her brand new Blamo! yo-yo. When she releases the yo-yo, it forms a 30 degree angle with the ground before coming back to her. If Allison holds the yo-yo above her head, it is 6 feet above the ground when she releases it. When the yo-yo touches the ground, how far away will it be from her feet?

A. 6 feet
B. $6\sqrt{2}$ feet
C. $6\sqrt{3}$ feet
D. $\frac{6}{\sqrt{3}}$ feet

13. If $f(x)=\frac{7}{4}x^4-x^3-\frac{15}{2}x^2+245x=38$,
at what value of x does concavity change?

A. x = -1
B. x = -5/7
C. x = 5/7
D. x = 1

14. Find the largest possible value of y if

$$y=\sqrt{\frac{3+\sin\left(\frac{x}{2}\right)}{25}}$$

A. $\frac{2}{5}$

B. $\frac{\sqrt{3}}{5}$

C. ∞

D. $\frac{\sqrt{5}}{5}$

15. Evaluate:

$$\log(0.01)+\left(\frac{\ln 18}{\ln 7}\right)(\log_3 1)+(\log_3 27)(\ln e)=?$$

A. -1
B. 0
C. 1
D. $\log_3 26$

16. IWU's gardeners plant money trees every year. It is estimated that if 20 trees are planted per acre, then each tree will yield $500 per year. For each additional tree planted per acre, the yield decreases by $5 per tree. What is the number of trees that should be planted per acre to receive the highest yield per year?

A. 75
B. 65
C. 60
D. 50

17. Find the following limit:

$$\lim_{x\to 3}\frac{x^3+3x^2-15x-9}{x^2-x-6}$$

A. 0
B. 6
C. 1
D. ∞

18. The derivative of a function f(x) can be found by using the following definition:

$$f'(x)=\lim_{h\to o}\frac{f(x+h)-f(x)}{h}$$

Use this definition to find the derivative of $f(x)=7x^2-3x+7$.

A. $14x+3$
B. $14x-4$
C. $7x^2-3$
D. $14x-3$

19. During a thunderstorm, a telephone pole 72 feet tall and perpendicular to the ground was broken by lightening. It's top landed on the ground 48 feet from the base of the pole. What are the lengths of the segments of the telephone pole?

A. 20 ft, 52 ft
B. 30 ft, 42 ft
C. 36 ft, 36 ft
D. 60 ft. 12 ft

20. In our number system, the base is 10. Seymour wants to know what the base 10 number 105 would be if he converted it to base 6. Can you give him the right answer?

A. 142
B. 386
C. 253
D. 455

21. If $28 = 3$ (in mod 5), and $18 = 4$ (in mod 7), solve for x: $47 = x$ (in mod 9).

A. 7
B. 5
C. 3
D. 2

22. Find the following limit:

$$\lim_{x\to\infty} \frac{x^5 - 3x + 7}{4x^5 - 6x^4 + 3x^2}$$

A. 0

B. $\frac{1}{4}$

C. 1

D. ∞

23. What is the last digit (ones place) of 3^{2004}.

A. 1
B. 3
C. 7
D. 9

24. How many integers n have both of the following properties:

1) $1 \leq n \leq 60$.
2) n and 60 have a common factor greater than 1.

A. 12
B. 16
C. 44
D. 48

25. How many points (x, y) on the graph of the circle $x^2 + y^2 = 25$ have the property that both x and y are integers?

A. None
B. 3
C. 8
D. 12