NCTM NCATE Standards (2012) – Secondary (Initial Preparation)

### Standard 1: Content Knowledge

Effective teachers of secondary mathematics demonstrate and apply knowledge of major mathematics concepts, algorithms, procedures, connections, and applications within and among mathematical content domains.

Preservice teacher candidates:

1a) Demonstrate and apply knowledge of major mathematics concepts, algorithms, procedures, applications in varied contexts, and connections within and among mathematical domains (Number, Algebra, Geometry, Trigonometry, Statistics, Probability, Calculus, and Discrete Mathematics) as outlined in the *NCTM NCATE Mathematics Content for Secondary*.

### Standard 2: Mathematical Practices

Effective teachers of secondary mathematics solve problems, represent mathematical ideas, reason, prove, use mathematical models, attend to precision, identify elements of structure, generalize, engage in mathematical communication, and make connections as essential mathematical practices. They understand that these practices intersect with mathematical content and that understanding relies on the ability to demonstrate these practices within and among mathematical domains and in their teaching.

Preservice teacher candidates:

2a) Use problem solving to develop conceptual understanding, make sense of a wide variety of problems and persevere in solving them, apply and adapt a variety of strategies in solving problems confronted within the field of mathematics and other contexts, and formulate and test conjectures in order to frame generalizations.

2b) Reason abstractly, reflectively, and quantitatively with attention to units, constructing viable arguments and proofs, and critiquing the reasoning of others; represent and model generalizations using mathematics; recognize structure and express regularity in patterns of mathematical reasoning; use multiple representations to model and describe mathematics; and utilize appropriate mathematical vocabulary and symbols to communicate mathematical ideas to others.

2c) Formulate, represent, analyze, and interpret mathematical models derived from real-world contexts or mathematical problems.

2d) Organize mathematical thinking and use the language of mathematics to express ideas precisely, both orally and in writing to multiple audiences.

2e) Demonstrate the interconnectedness of mathematical ideas and how they build on one another and recognize and apply mathematical connections among mathematical ideas and across various content areas and real-world contexts.

2f) Model how the development of mathematical understanding within and among mathematical domains intersects with the mathematical practices of problem solving, reasoning, communicating, connecting, and representing.
Standard 3: Content Pedagogy
Effective teachers of secondary mathematics apply knowledge of curriculum standards for mathematics and their relationship to student learning within and across mathematical domains. They incorporate research-based mathematical experiences and include multiple instructional strategies and mathematics-specific technological tools in their teaching to develop all students’ mathematical understanding and proficiency. They provide students with opportunities to do mathematics – talking about it and connecting it to both theoretical and real-world contexts. They plan, select, implement, interpret, and use formative and summative assessments for monitoring student learning, measuring student mathematical understanding, and informing practice.

Preservice teacher candidates:
3a) Apply knowledge of curriculum standards for secondary mathematics and their relationship to student learning within and across mathematical domains.
3b) Analyze and consider research in planning for and leading students in rich mathematical learning experiences.
3c) Plan lessons and units that incorporate a variety of strategies, differentiated instruction for diverse populations, and mathematics-specific and instructional technologies in building all students’ conceptual understanding and procedural proficiency.
3d) Provide students with opportunities to communicate about mathematics and make connections among mathematics, other content areas, everyday life, and the workplace.
3e) Implement techniques related to student engagement and communication including selecting high quality tasks, guiding mathematical discussions, identifying key mathematical ideas, identifying and addressing student misconceptions, and employing a range of questioning strategies.
3f) Plan, select, implement, interpret, and use formative and summative assessments to inform instruction by reflecting on mathematical proficiencies essential for all students.
3g) Monitor students’ progress, make instructional decisions, and measure students’ mathematical understanding and ability using formative and summative assessments.
Standard 4: Mathematical Learning Environment
Effective teachers of secondary mathematics exhibit knowledge of adolescent learning, development, and behavior. They use this knowledge to plan and create sequential learning opportunities grounded in mathematics education research where students are actively engaged in the mathematics they are learning and building from prior knowledge and skills. They demonstrate a positive disposition toward mathematical practices and learning, include culturally relevant perspectives in teaching, and demonstrate equitable and ethical treatment of and high expectations for all students. They use instructional tools such as manipulatives, digital tools, and virtual resources to enhance learning while recognizing the possible limitations of such tools.

Preservice teacher candidates:
4a) Exhibit knowledge of adolescent learning, development, and behavior and demonstrate a positive disposition toward mathematical processes and learning.
4b) Plan and create developmentally appropriate, sequential, and challenging learning opportunities grounded in mathematics education research in which students are actively engaged in building new knowledge from prior knowledge and experiences.
4c) Incorporate knowledge of individual differences and the cultural and language diversity that exists within classrooms and include culturally relevant perspectives as a means to motivate and engage students.
4d) Demonstrate equitable and ethical treatment of and high expectations for all students.
4e) Apply mathematical content and pedagogical knowledge to select and use instructional tools such as manipulatives and physical models, drawings, virtual environments, spreadsheets, presentation tools, and mathematics-specific technologies (e.g., graphing tools, interactive geometry software, computer algebra systems, and statistical packages); and make sound decisions about when such tools enhance teaching and learning, recognizing both the insights to be gained and possible limitations of such tools.
Standard 5: Impact on Student Learning
Effective teachers of secondary mathematics provide evidence demonstrating that as a result of their instruction, secondary students’ conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and application of major mathematics concepts in varied contexts have increased. These teachers support the continual development of a productive disposition toward mathematics. They show that new student mathematical knowledge has been created as a consequence of their ability to engage students in mathematical experiences that are developmentally appropriate, require active engagement, and include mathematics-specific technology in building new knowledge.

Preservice teacher candidates:
5a) Verify that secondary students demonstrate conceptual understanding; procedural fluency; the ability to formulate, represent, and solve problems; logical reasoning and continuous reflection on that reasoning; productive disposition toward mathematics; and the application of mathematics in a variety of contexts within major mathematical domains. 
5b) Engage students in developmentally appropriate mathematical activities and investigations that require active engagement and include mathematics-specific technology in building new knowledge. 
5c) Collect, organize, analyze, and reflect on diagnostic, formative, and summative assessment evidence and determine the extent to which students’ mathematical proficiencies have increased as a result of their instruction.

Standard 6: Professional Knowledge and Skills
Effective teachers of secondary mathematics are lifelong learners and recognize that learning is often collaborative. They participate in professional development experiences specific to mathematics and mathematics education, draw upon mathematics education research to inform practice, continuously reflect on their practice, and utilize resources from professional mathematics organizations.

Preservice teacher candidates:
6a) Take an active role in their professional growth by participating in professional development experiences that directly relate to the learning and teaching of mathematics. 
6b) Engage in continuous and collaborative learning that draws upon research in mathematics education to inform practice; enhance learning opportunities for all students’ mathematical knowledge development; involve colleagues, other school professionals, families, and various stakeholders; and advance their development as a reflective practitioner. 
6c) Utilize resources from professional mathematics education organizations such as print, digital, and virtual resources/collections.
## Standard 7: Secondary Mathematics Field Experiences and Clinical Practice
Effective teachers of secondary mathematics engage in a planned sequence of field experiences and clinical practice under the supervision of experienced and highly qualified mathematics teachers. They develop a broad experiential base of knowledge, skills, effective approaches to mathematics teaching and learning, and professional behaviors across both middle and high school settings that involve a diverse range and varied groupings of students. Candidates experience a full-time student teaching/internship in secondary mathematics directed by university or college faculty with secondary mathematics teaching experience or equivalent knowledge base.

Preservice teacher candidates:

**7a)** Engage in a sequence of planned field experiences and clinical practice prior to a full-time student teaching/internship experience that include observing and participating in both middle and high school mathematics classrooms and working with a diverse range of students individually, in small groups, and in large class settings under the supervision of experienced and highly qualified mathematics teachers in varied settings that reflect cultural, ethnic, linguistic, gender, and learning differences.

**7b)** Experience full-time student teaching/internship in secondary mathematics that is supervised by a highly qualified mathematics teacher and a university or college supervisor with secondary mathematics teaching experience or equivalent knowledge base.

**7c)** Develop knowledge, skills, and professional behaviors across both middle and high school settings; examine the nature of mathematics, how mathematics should be taught, and how students learn mathematics; and observe and analyze a range of approaches to mathematics teaching and learning, focusing on tasks, discourse, environment, and assessment.
**A. Secondary Mathematics Teachers**

All secondary mathematics teachers should be prepared with depth and breadth in the following mathematical domains: Number, Algebra, Geometry, Trigonometry, Statistics, Probability, Calculus, and Discrete Mathematics. All teachers certified in secondary mathematics should know, understand, teach, and be able to communicate their mathematical knowledge with the breadth of understanding reflecting the following competencies for each of these domains.

**A.1. Number and Quantity**

To be prepared to develop student mathematical proficiency, all secondary mathematics teachers should know the following topics related to number and quantity with their content understanding and mathematical practices supported by appropriate technology and varied representational tools, including concrete models:

A.1.1 Structure, properties, relationships, operations, and representations including standard and non-standard algorithms, of numbers and number systems including integer, rational, irrational, real, and complex numbers

A.1.2 Fundamental ideas of number theory (divisors, factors and factorization, primes, composite numbers, greatest common factor, least common multiple, and modular arithmetic)

A.1.3 Quantitative reasoning and relationships that include ratio, rate, and proportion and the use of units in problem situations

A.1.4 Vector and matrix operations, modeling, and applications

A.1.5 Historical development and perspectives of number, number systems, and quantity including contributions of significant figures and diverse cultures

**A.2. Algebra**

To be prepared to develop student mathematical proficiency, all secondary mathematics teachers should know the following topics related to algebra with their content understanding and mathematical practices supported by appropriate technology and varied representational tools, including concrete models:

A.2.1 Algebraic notation, symbols, expressions, equations, inequalities, and proportional relationships, and their use in describing, interpreting, modeling, generalizing, and justifying relationships and operations

A.2.2 Function classes including polynomial, exponential and logarithmic, absolute value, rational, and trigonometric, including those with discrete domains (e.g., sequences), and how the choices of parameters determine particular cases and model specific situations
A.2.3 Functional representations (tables, graphs, equations, descriptions, recursive definitions, and finite differences), characteristics (e.g., zeros, intervals of increase or decrease, extrema, average rates of change, domain and range, and end behavior), and notations as a means to describe, reason, interpret, and analyze relationships and to build new functions

A.2.4 Patterns of change in linear, quadratic, polynomial, and exponential functions and in proportional and inversely proportional relationships and types of real-world relationships these functions can model

A.2.5 Linear algebra including vectors, matrices, and transformations

A.2.6 Abstract algebra, including groups, rings, and fields, and the relationship between these structures and formal structures for number systems and numerical and symbolic calculations

A.2.7 Historical development and perspectives of algebra including contributions of significant figures and diverse cultures

A.3. Geometry and Trigonometry

To be prepared to develop student mathematical proficiency, all secondary mathematics teachers should know the following topics related to geometry and trigonometry with their content understanding and mathematical practices supported by appropriate technology and varied representational tools, including concrete models:

A.3.1 Core concepts and principles of Euclidean geometry in two and three dimensions and two-dimensional non-Euclidean geometries

A.3.2 Transformations including dilations, translations, rotations, reflections, glide reflections; compositions of transformations; and the expression of symmetry in terms of transformations

A.3.3 Congruence, similarity and scaling, and their development and expression in terms of transformations

A.3.4 Right triangles and trigonometry

A.3.5 Application of periodic phenomena and trigonometric identities

A.3.6 Identification, classification into categories, visualization, and representation of two- and three-dimensional objects (triangles, quadrilaterals, regular polygons, prisms, pyramids, cones, cylinders, and spheres)

A.3.7 Formula rationale and derivation (perimeter, area, surface area, and volume) of two- and three-dimensional objects (triangles, quadrilaterals, regular polygons, rectangular prisms,
pyramids, cones, cylinders, and spheres), with attention to units, unit comparison, and the iteration, additivity, and invariance related to measurements

A.3.8 Geometric constructions, axiomatic reasoning, and proof

A.3.9 Analytic and coordinate geometry including algebraic proofs (e.g., the Pythagorean Theorem and its converse) and equations of lines and planes, and expressing geometric properties of conic sections with equations

A.3.10 Historical development and perspectives of geometry and trigonometry including contributions of significant figures and diverse cultures

A.4. Statistics and Probability
To be prepared to develop student mathematical proficiency, all secondary mathematics teachers should know the following topics related to statistics and probability with their content understanding and mathematical practices supported by appropriate technology and varied representational tools, including concrete models:

A.4.1 Statistical variability and its sources and the role of randomness in statistical inference

A.4.2 Creation and implementation of surveys and investigations using sampling methods and statistical designs, statistical inference (estimation of population parameters and hypotheses testing), justification of conclusions, and generalization of results

A.4.3 Univariate and bivariate data distributions for categorical data and for discrete and continuous random variables, including representations, construction and interpretation of graphical displays (e.g., box plots, histograms, cumulative frequency plots, scatter plots), summary measures, and comparisons of distributions

A.4.4 Empirical and theoretical probability (discrete, continuous, and conditional) for both simple and compound events

A.4.5 Random (chance) phenomena, simulations, and probability distributions and their application as models of real phenomena and to decision making

A.4.6 Historical development and perspectives of statistics and probability including contributions of significant figures and diverse cultures

A.5. Calculus
To be prepared to develop student mathematical proficiency, all secondary mathematics teachers should know the following topics related to calculus with their content understanding and mathematical practices supported by appropriate technology and varied representational tools, including concrete models:
A.5.1 Limits, continuity, rates of change, the Fundamental Theorem of Calculus, and the meanings and techniques of differentiation and integration

A.5.2 Parametric, polar, and vector functions

A.5.3 Sequences and series

A.5.4 Multivariate functions

A.5.5 Applications of function, geometry, and trigonometry concepts to solve problems involving calculus

A.5.6 Historical development and perspectives of calculus including contributions of significant figures and diverse cultures

A.6. Discrete Mathematics
To be prepared to develop student mathematical proficiency, all secondary mathematics teachers should know the following topics related to discrete mathematics with their content understanding and mathematical practices supported by appropriate technology and varied representational tools, including concrete models:

A.6.1 Discrete structures including sets, relations, functions, graphs, trees, and networks

A.6.2 Enumeration including permutations, combinations, iteration, recursion, and finite differences

A.6.3 Propositional and predicate logic

A.6.4 Applications of discrete structures such as modeling and solving linear programming problems and designing data structures

A.6.5 Historical development and perspectives of discrete mathematics including contributions of significant figures and diverse cultures