Math 200, Spring 2009

Techniques of Proof

2 MWF, CNS E108

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Required Texts:	Leslie Lamport
	The LaTeX User's Guide and Reference Manual
	2nd Edition

Course Description

This is a course on how to write mathematical proofs. Since most 300 level math courses and all 400 level ones use a lot of proof and expect you to be able to find and write proofs it serves as one of the keystones of our curriculum. What you will do in this course is search for proofs (both alone in your homework and with your peers in class), critique proposed proofs to see if they are correct, write the proofs you have found, rewrite what you have written so that it is clear and expressed in (what passes in mathematics for) flowing English. In addition you will learn how to typeset your work in LaTeX, the mark-up language used for producing mathematical text. All of your grade in this course will be earned by finding and writing proofs.

This course is writing intensive, not just because you need a writing intensive course in the major to graduate, but also because the process of finding a proof and the process of writing it down coherently are best learned by doing, not watching. We come to know mathematics through a many staged process: first we explore examples; then make conjectures, look for counterexamples, and if none are found search for a proof; eventually we see the idea of a proof; then we fill in all of the details to be certain of the result, often finding flaws which we need to correct; then we write the proof so that the idea of the proof is clear, there are enough details given so that the certainty of the result is not in doubt, and the argument flows. You will

often only find the flaws in your own reasoning when you attempt to write the final version of a proof or present the proof to a peer. The argument in a proof in mathematics follows rules which have been codified in (usually) classical logic as forms of argument which will not introduce error. While the search for a proof may follow experimental, computational, or diagrammatic approaches and may make leaps of faith which leave holes to be filled in later, the exposition of a proof takes a deductive approach, starting from definitions and previously proved results and arguing to the result being proved.

We will start by looking at the major techniques of proof and how they arise from standard ideas in logic. We will look at standard strategies in proof: working forward; working backwards; seeing what general features you can see in a good example. We will look at the grammar of mathematical symbolism and how mathematics gets incorporated into text.

The mathematical content of the course comes from naive set theory, number theory, category theory, geometry, and calculus. Most of modern mathematics takes set theory as a starting place and makes its definitions in set theoretic terms. In our curriculum this is most apparent in probability, topology, and modern algebra. The point of the course, though, is for you to develop skill in finding and writing proofs rather than coverage of any particular mathematical content.

Written work and grading:

In this class you will find proofs, give me rough drafts, and then polish the results. You will accumulate points for this activity. There will also be an expository paper with 100 points possible. If you get 500 points I'll guarantee an A. The minimum for a C is a consistent struggle leading to marked improvement but perhaps only 250 points.

I will provide several assignment packets giving the basic definitions in an area and a collection of propositions with the instructions to give proof, or to give a counter example and a salvage of the proposition with a proof. After I hand out the packet you'll have about two weeks to get rough drafts (each proof on a separate page) in. If at first you don't succeed, keep trying. A reasonably common grade on a rough draft is "not yet". Final drafts typeset in LaTeXare due one week after I return the approved rough draft. Each theorem, proposition or problem has a point value on the sheet (averaging about 3 points); a correct proof in rough draft earns full credit. If what you

hand in isn't a correct proof I'll make comments and ask you to try again. Don't expect to stop there. I will ask for revision into a polished, airtight, typeset version. Hand in the rough draft stapled to the final version so I can see how you worked on the exposition. The final draft is worth up to the same number of points as the rough draft.

Proving a result is making it your own, not because it is original but because once you have proved something you *really know* it. When you find a proof of one of the results (ideally on your own, but there are books and research papers out there with the proofs of these results in them and some proofs you will work on in class as part of a group–mathematicians do occasionally talk to each other about their partial results, though mathematics is a much more solitary activity than science or computer programming) write a draft and turn it in to me. If you got the basic idea for a proof from another source, give credit to that source. The exposition should be your own.

There will also be one expository paper. It should have all sources properly credited. It will be due Monday, February 16.

At the time set for the final I will ask each of you to pick a proof you think is your best work and present the polished version to the class.

Attendance Policy

Classes and office hours are what you pay tuition for, so take advantage of them. If you don't come to class you will not learn the material with the same emphasis that I put on it. I do not deduct points for classes missed.

Policy on Academic Integrity

I want to see your own work: any writing you hand in should be in your own words expressing reasoning you have made your own. If you got the basic idea for a proof from another source, give credit to that source. In the assignment of the expository paper we will discuss how mathematicians cite other people's work. Some other sections of this course (and some of the 400 level courses here) use the Moore method, which has much stricter limitations on what you may refer to.