

Math 110, Finite Math, Spring 2007
Midterm Exam

Name:

Work all problems.

1. A manufacturer of pennywhistles has fixed costs of \$150 and variable cost of \$0.50 per whistle. Whistles sell for \$8.00 each. How many must be sold to break even? 10 Points

2. Give an equation for the line perpendicular to $3x + 4y = 5$ through the point (1,2). 10 Points

3. Set up the augmented matrix for the system 10 Points

$$\begin{aligned}2x - 3y &= z + x - 3 \\y - 2x + z &= 4 \\3 - x &= y + 4z\end{aligned}$$

4. The following matrices are the last step in solving a system of equations for the variables x_1, x_2, x_3, x_4 . What do they mean? 5 Points Each

(a)
$$\begin{pmatrix} 1 & 0 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 6 \\ 0 & 0 & 0 & 1 & 10 \end{pmatrix}$$

(b)
$$\begin{pmatrix} 1 & 2 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

(c)
$$\begin{pmatrix} 1 & 2 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 3 \end{pmatrix}$$

5. Do the next step in Gauss-Jordan elimination. 5 Points Each

(a)
$$\begin{pmatrix} 1 & 2 & 0 & 0 & 2 \\ 0 & 2 & 4 & 0 & 8 \\ 0 & 3 & 1 & 1 & 2 \\ 0 & 7 & 1 & 0 & 3 \end{pmatrix}$$

(b)
$$\begin{pmatrix} 1 & 2 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 2 & 0 & 1 & 2 \\ 0 & 0 & 1 & 1 & 4 \end{pmatrix}$$

(c)
$$\begin{pmatrix} 1 & 2 & 0 & 0 & 2 \\ 1 & 3 & 1 & 0 & 3 \\ 2 & 0 & 1 & 1 & 2 \\ 1 & 3 & 4 & 5 & 3 \end{pmatrix}$$

6. Find

10 Points

$$\begin{pmatrix} 1 & -3 & 2 \\ 0 & 1 & 3 \end{pmatrix} \begin{pmatrix} 3 & -1 \\ 1 & 0 \\ 4 & -5 \end{pmatrix} + 3 \begin{pmatrix} 1 & -2 \\ -3 & 4 \end{pmatrix}$$

7. An economy has three sectors: agriculture, energy, and transportation. To produce each unit of agricultural production the agricultural sector consumes .1 units of agriculture, .2 units of transportation, and .1 unit of energy. To produce a unit of energy requires .5 units of transportation and .3 units of agriculture. To produce a unit of transportation requires .2 units of agriculture and .3 units of energy. How many units of each sector's output are used to produce 100 units of agriculture, 200 units of transportation, and 80 units of energy?

15 Points

8. Find the inverse of $\begin{pmatrix} 1 & 0 & 2 \\ 2 & 2 & 4 \\ 1 & 2 & 3 \end{pmatrix}$

15 Points

9. Here is a linear programming problem:

Maximize $10x + 3y$ subject to

$$x + 4y \leq 20$$

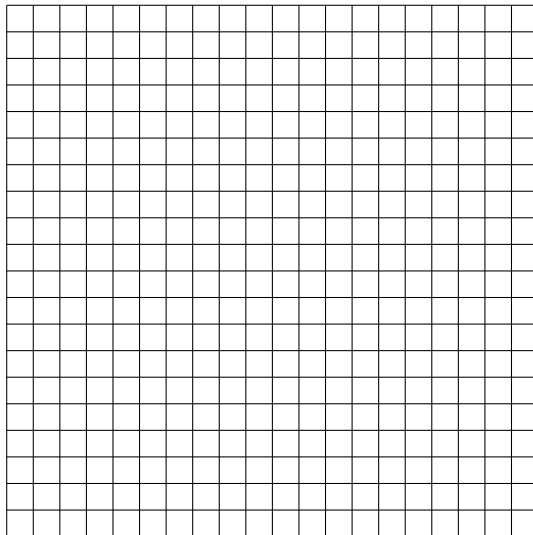
$$2x + y \leq 12$$

$$x + y \geq 3$$

$$x \geq 0 \quad y \geq 0$$

(a) Sketch the feasible region and label its corners.

10 Points



(b) Where does the maximum happen?

5 Points

10. Here is a tableau from the middle of a simplex algorithm in which the non-slack variables were x_1, x_2, x_3 :

$$\left(\begin{array}{cccccc|c} 1 & 2 & 0 & 1 & 0 & 1 & 0 & 7 \\ 0 & 1 & 1 & 2 & 0 & 2 & 0 & 4 \\ 0 & -1 & 0 & 3 & 0 & 3 & 1 & 2 \\ 0 & 3 & 0 & -1 & 1 & 4 & 0 & 9 \\ \hline 0 & -6 & 0 & -4 & 0 & 8 & 0 & 15 \end{array} \right)$$

5 Points Each

- (a) What are the values of x_1, x_2 , and x_3 ?
- (b) What is the value of the objective?
- (c) Is this point in the feasible region?
- (d) Is this solution optimal?
- (e) Circle the next pivot.

11. Do a pivot on the circled entry:

10 Point

$$\left(\begin{array}{cccc|c} 2 & 0 & 0 & 4 & 1 & 8 \\ 3 & 1 & 0 & 2 & 0 & 13 \\ 1 & 0 & 1 & 2 & 0 & 5 \\ \hline -2 & 0 & 0 & -3 & 0 & 9 \end{array} \right)$$