Example

A credit card charges an annual rate of 14% compounded monthly. This month's bill is \$6000. The minimum payment is \$125. Suppose I keep paying \$125 each month. How long will it take to pay off the bill? What is the total interest paid during that period?

Here we want n so that

$$6000 = 125 \frac{1 - \left(1 + \frac{.14}{12}\right)^{-n}}{\frac{.14}{12}}$$

So

$$n = -\frac{\log\left(1 - \frac{6000}{125}\frac{.14}{12}\right)}{\log\left(1 + \frac{.14}{12}\right)} = 70.7795$$

So, about 71 months.
The interest paid is

$$71 \times 125 - 6000 = 8875 - 6000 = 2875$$

Example

Suppose you want an item which costs \$10000. You can save for it at \$200 per month in an account earning 3% interest compounded monthly, or you can borrow at 8% paying off at a rate of \$200 per month. With both of these options how long will it take until you have finished the payments? What is the total paid in each case?

First let's analyze the savings plan: We need to find n so that

$$10000 = 200\frac{(1+\frac{.03}{12})^n - 1}{\frac{.03}{12}}$$

So

$$n = \frac{\log\left(50\frac{.03}{12}+1\right)}{\log(1+\frac{.03}{12})} = 47.12$$

Rounding up gives $n = 48$ for a total of \$200n =\$9600

For pure loan: We need n so that

$$10000 = 200\frac{1-\left(1+\frac{.08}{12}\right)^{-n}}{\frac{.08}{12}}$$

so

$$n = -\frac{\log(1-50\frac{.08}{12})}{\log\left(1+\frac{.08}{12}\right)} \approx 61$$

So the total of the payments is $61 \times \$200 = \$12200$.

Example (continued)

Suppose you save for a year and then take out the loan for the balance. How much do you end up paying?

Here we first figure out how much gets saved:

$$S = 200\frac{\left(1+\frac{.03}{12}\right)^{12}-1}{\frac{.03}{12}} = 2433.28$$

So we need to borrow $\$10000 - \$2433.28 = \$7566.72$ That will take

$$n = -\frac{\log(1-\frac{7566.72}{200}\frac{.08}{12})}{\log\left(1+\frac{.08}{12}\right)} \approx 44$$

payments, so the total paid will be $56 \times \$200 = \$11,200$.

Example

A car dealer offers either 0% financing for 3 years or a \$3000 rebate on a \$25000 car. Assuming that you could find a loan for 6% for 36 months which deal gives you a smaller payment?

The \$25000 financed at 0% will have a payment of $\$25000/36 = 694.45$.
If you finance $\$25000 - \$3000$ at 6% your payments will be

$$R = \frac{22000 \times \frac{.06}{12}}{1 - \left(1 + \frac{.06}{12}\right)^{-36}} = 669.283$$

Example

A 30 year mortgage on a loan of $150000 at 5.25% has been in force for a few years. The homeowner needs to know how much interest was paid in the ninth year of the loan. Figure this out.

First we find out what the payment each month is:

$$R = \frac{150000 \times \frac{.0525}{12}}{\left(1 - \left(1 + \frac{.0525}{12}\right)^{-360}\right)} = 828.31$$

The easy thing to find is how much of the loan has been paid off by taking the present value of a loan with 22 years of payments and subtracting the present value of a loan with 21 years of payments:

$$828.31\left(\frac{\left(1 - \left(1 + \frac{.0525}{12}\right)^{-264}\right) - \left(1 - \left(1 + \frac{.0525}{12}\right)^{-252}\right)}{\frac{.0525}{12}}\right) = 3616.14$$

So the interest paid is $12 \times 828.31 - 3616.14 = 6323.58$.

Example

When you take out a large loan you are often given an amortization schedule which shows how much of each payment is interest and how much is going to reduce the principle. This is easily derived on a spreadsheet. Let's do it for a 15 year loan of \$100000 at 5.8% interest.

This is best done on a spreadsheet. First we find the payment. The interest portion of that payment is the simple interest for one month on the unpaid balance at the beginning of that month. The rest of the payment goes to reduce the principle. Repeat, using the fill feature of the spreadsheet.

Example

A credit card company charges .0325% on the average daily balance for old purchases. If my current balance is $6000 and I pay the minimum each month (let's say 2% of the outstanding balance) finally paying off the total when it gets less than $100, how much will I end up paying in total?

This is best computed using a spreadsheet giving the payment made, then subtracting the interest on the unpaid balance, and then reducing the balance by what remains of the payment. Fill, then see when the loan gets paid off. In class we did this on Excell and it took on the order of 100 years!

A couple we didn't get to in class:

Example

If you manage to accumulate $500000 at retirement and decide to put it in an account earning 6% nominal interest and want to make regular monthly withdrawals forever without running out of money how big can your withdrawals be?
What if you just want to be able to make the withdrawals for 35 years?

Example

A homeowner takes out a loan for \$250000 on which interest rate is 4% compounded monthly. The payment is determined by assuming that the loan would be for 30 years, but the loan actually has a balloon payment due after 5 years for the remaining balance. One way to arrange for that balloon payment is to set up a sinking fund by saving regularly to accumulate the needed amount. If you can get 6% interest on the savings account, how much will the total payments (for the loan plus the sinking fund) be each month?