Review sheet for Exam 1, Calculus 1

September 26, 2008

1 Definitions and their uses

We have spent most of the year so far making explicit use of the δ-ϵ definition of $\lim_{x \to a} f(x) = L$. Expect to be asked for that definition and to use it to show (prove) that a particular function has a given limit.

Definition 1 *For a function defined near, but not necessarily at a, $\lim_{x \to a} f(x) = L$ means that for any $\epsilon > 0$ there is a δ such that for all x if $0 < |x - a| < \delta$ then $|f(x) - L| < \epsilon$.*

We also used the definition of continuity for functions extensively.

Definition 2 *The function f is continuous at a if and only if $\lim_{x \to a} f(x) = f(a)$.*

We also had definitions for $\lim_{x \to \infty} f(x) = L$, $\lim_{x \to a} f(x) = \infty$, $\lim_{x \to a^+} f(x) = L$, and $\lim_{x \to a^-} f(x) = L$.

We used the definition explicitly to show that the following functions are continuous:

1. $f(x) = \sqrt{x}$ for positive values
2. $f(x) = \sqrt[3]{x}$ everywhere
3. $f(x) = x^3$
4. Linear functions
5. the sine and cosine functions (using the Squeeze theorem)
6. polynomial functions (using sum, constant multiple, and product theorems)

2 Major theorems

Some theorems you should be able to prove:

1. (Limit of sums) If $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$ then $\lim_{x\to a} f(x) + g(x) = L + M$
2. (Constant multiples) If $\lim_{x\to a} f(x) = L$ then $\lim_{x\to a} kf(x) = kL$ for any constant k.
3. (Squeeze Theorem) If $f(x) \leq g(x) \leq h(x)$ and $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} h(x) = L$, then $\lim_{x\to a} g(x) = L$.
4. (Removing holes) If $f(x) = g(x)$ except at a and $\lim_{x\to a} f(x) = L$ then $\lim_{x\to a} g(x) = L$.
5. (One sided limits) If $\lim_{x\to a^+} f(x) = L$ and $\lim_{x\to a^-} f(x) = L$, then $\lim_{x\to a} f(x) = L$.
6. (Moving limits inside continuous functions): If f is continuous at M and $\lim_{x\to a} g(x) = M$ then $\lim_{x\to a} f \circ g(x) = f(M)$.

Some additional ones you should be able to use explicitly:

1. (Limit of products) If $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$ then $\lim_{x\to a} f(x)g(x) = LM$.
2. (Limit of quotients) If $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M \neq 0$ then $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{L}{M}$.
3. (Basic trig limits) $\lim_{x\to 0} \frac{\sin(x)}{x} = 1$ and $\lim_{x\to 0} \frac{1-\cos(x)}{x} = 0$.

You should also know what the theorems on Conservation of Sign, the Intermediate Value Theorem, and the Extreme Value Theorem tell you.

3 Technique

For some functions you should be able to give a δ-ϵ proof that a limit has a particular value.

For more complex functions you should be able to find limits using the limit theorems explicitly. Some problems will also be given where I do not ask for explicit justification.